QUARK-INDUCED CORRELATIONS BETWEEN INSTANTONS DRIVE THE CHIRAL PHASE TRANSITION

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February 7, 2008

Abstract

A simple model for the instanton ensemble at finite temperature T is proposed, including "random" and strongly correlated "molecular" component. T-dependence of fermionic zero modes naturally leads to chiral symmetry restoration, without instanton suppression. Moreover, at $T = (1-2)T_c$ the non-perturbative effects due to "molecules" are so strong, that they even dominate the global thermodynamics.

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Instantons are the major component of non-perturbative fields in the QCD vacuum, and significant amount of work [1, 2, 3] has been done in order to build a quantitative theory describing them. Two major steps forward were done during the last year. First, the simplest ensemble (the so called Random Instanton Liquid Model), has reproduced many correlation functions [4] known from phenomenology [5] and lattice simulations [6]. Second, by "cooling" of the (quenched) lattice configurations it was found in [7] that the typical instanton density is about $n \approx 1.4 fm^{-4}$ and the typical size is about $\rho \approx .35 fm$, very close to the "instanton liquid" parameters suggested by one of us a decade ago [2]. The correlation functions and hadronic wave functions, are shown to be practically unaffected by "cooling": so, by removing *perturbative gluons* and *confinement*, one does not loose mesons and baryons!

The instanton-induced effects at finite temperature T and much less studied. Our particular focus is on the chiral restoration phase transition at T_c. The main idea is that it happens due to growing correlations between instantons and anti-instantons, described by the disappearance of "single" instantons and growth of $\bar{I}I$ "molecules".

Our first study along this line was performed few years ago [8]. However, the present paper is significantly different, because it is based on completely new mechanism. Instead of Debye-type screening [9], implemented by *thermal suppression factor* $f(T) = exp(-\rho^2 T^2 const)$ [10] [1] , the phase transition occurs due to the T-dependence of the *quark-induced* $\bar{I}I$ interaction.

The ensemble of interacting instantons can be studied with the partition func-

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[1]This suppression is expected to work at *high* $T >> T_c$ only, and available lattice data on screening mass suggest that it may probably happens at only at temperatures $T > 300 - 400 MeV$. Direct lattice measurements of the instanton density [11] have indeed seen no significant T dependence till such high temperatures.

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$$Z = \int \sum_{N_+ N_-} \frac{1}{N_+!N_-!} \prod_i^{N_++N_-} d\Omega_i d(\rho_i)\rho_i^{N_f} exp(-S_{int})\Pi_i^{N_f} det(i\hat{D} + im_f) \qquad (1)$$

where $d\Omega_i$ is the measure in space of collective coordinates, (12 per instanton in $SU(3)$), $d(\rho)$ the instanton amplitude, and S_{int} the gluonic interaction. The last factor, appearing after integration over fermions, is the one we are focussing on. Assuming $N_+ = N_- = N$, one writes it in terms of a $N \times N$ 'hopping' matrix, with $T_{IA} = \int d^4x \langle \psi_{A0}(x-z_A)|i\hat{D}_x|\psi_{I0}(x-z_I)\rangle$ from some instanton I to some anti-instanton A (z_A, z_I are the centers of A, I, ψ_0 the zero-modes), as $det(TT^+ + m_f^2)$. The statistical system described by this partition function is quite complicated, and direct simulations were done so far only for $T = 0$.

The instanton solutions and their zero modes are known analytically for non-zero T, and detailed studies of 'hopping' matrix elements were done in [12]. They have the structure $T_{IA} = u_4 f_1 + (\mathbf{ur}/r)f_2$, where the 2×2 matrix $u_\mu\tau_\mu^+$ describes the relative orientation of I and A. Rather complicated formulae for f_1, f_2 were derived in [12].

We consider the instanton ensemble as a superposition of the uncorrelated (or "random'") component and a highly correlated (or "molecular") component, with the (4-d) densities $n_a(T), n_m(T)$. In the former component instantons are assumed to have *random* relative orientation u_μ, and, as in [8], it is treated in a mean field approximation. The fermionic determinant is obtained from $I(T) \sim \langle T_{IA}T_{AI}^+\rangle$, *i.e.* summed over anti-instantons A with a density $n_a/2$, with random *positions* and *orientations*.

For the "molecular" component we assume the opposite, namely the most favorable relative color orientation $u_\mu \sim (z_I^\mu - z_A^\mu)$, maximizing the hopping matrix element $|T_{IA}|$ (as well as $exp(-S_{int})$). Therefore, we have to calculate another function $\tilde{I}(N_f, T) \sim \langle (T_{IA}T_{AI}^*)^{N_f}\rangle$ where now averaging means integration only over the *relative coordinates* inside a $\bar{I}I$ pair.

Temperature dependence of these average matrix elements was obtained by numerical integration of the formulae from [12], and the results are shown in Fig. 1. One has to exclude too close $\bar{I}I$ pairs[2] , and we show results with the "core radii" $R_c = 1\rho$ (dashed) and $R_c = 2\rho$ (solid). Although the overlap integrals significantly depend on R_c, the resulting uncertainty of the thermodynamical quantities (see below) is in fact not so dramatic. Note remarkably different T-dependence of these two quantities. While the $I(T)$ (indicated as the $N_f = 1$ curves in Fig.1) *decreases* with T, $\tilde{I}(N_f, T)$ *grows*, the stronger the larger N_f. As a result, the molecular component builds up with T, while the random one decreases and eventually disappears, restoring the chiral symmetry.

In order to simplify gluonic interaction, and still describe the system self-consistently, we adopt a simple "average repulsion" $\langle S_{int} \rangle = \kappa \rho_I^2 \rho_A^2$ with one dimensionless parameter κ. The same parametrization is used for the description of interaction between all instantons, belonging to the random component or to molecules.

Let us now evaluate the statistical sum of the system in terms of the densities n_a, n_m, starting with the differential activities for molecular

$$dz_m = C^2 d\rho_1 d\rho_2 d^4 R \frac{d\Omega_{SU(3)}}{\Omega_{SU(3)}} (\rho_1 \rho_2)^{b-5} exp[-\kappa(\rho_1^2 + \rho_2^2)(\bar{\rho}_a^2 n_a + 2\bar{\rho}_m^2 n_m)](T_{IA}T_{AI}^*)^{N_f} \quad (2)$$

and random components

$$dz_a = 2C d\rho \rho^{b-5} exp[-\kappa \rho^2(\bar{\rho}_a^2 n_a + 2\bar{\rho}_m^2 n_m)] < TT^+ >^{N_f} \quad (3)$$

where $b = \frac{11}{3}N_c - \frac{2}{3}N_f$ is the coefficient of the Gell-Mann-Low function and $< TT^+ >= \rho^{3/2}[\frac{1}{2}I(T)\int dn_a(\rho)\rho]^{1/2}$. Besides the densities, an important ingredient of the interaction are the root mean square radii $\bar{\rho}_a, \bar{\rho}_m$, which can be found from

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[2]The general reason for such 'repulsive core' is generally related to the fact, that too close pairs do not in fact correspond to strong fields and are not objects of the semiclassical theory.

eqs. (3) and (4) to be related to each other through

$$\frac{\bar{\rho}_m^2}{\bar{\rho}_a^2} = \frac{\alpha}{\beta}; \quad \alpha = b/2 - 1; \quad \beta = b/2 + 3N_f/4 - 2 \tag{4}$$

Another relation between them connects the interaction parameter κ to the diluteness of the ensemble

$$\frac{1}{\kappa} = \frac{2\bar{\rho}_a^4 n_a}{\beta} + \frac{4\bar{\rho}_m^4 n_m}{\alpha} \quad . \tag{5}$$

So, one can eliminate the mean square radii and get the activities

$$z_m = \frac{A}{[n_a + (2\alpha/\beta)n_m]^\alpha}; \quad A = \frac{\tilde{I}(N_f, T)C^2\Gamma^2(\alpha)}{(4\kappa\beta)^\alpha} \tag{6}$$

$$z_a = \frac{Bn_a^{N_f/2}}{[n_a + (2\alpha/\beta)n_m]^{\beta/2+N_f/8}}; \quad B = \frac{C\Gamma(\beta)}{(2\kappa)^\beta}\left(\frac{I(T)}{2}\right)^{N_f/2}\left(\frac{\beta}{\kappa}\right)^{N_f/8-\beta/2} \quad . \tag{7}$$

As usual, the grand potential

$$\Omega = -p = -(logZ)/V_4 = \frac{N_a}{V_4}log\left(\frac{ez_aV_4}{N_a}\right) + \frac{N_m}{V_4}log\left(\frac{ez_mV_4}{N_m}\right) \tag{8}$$

(where V_4 is the 4-dimensional volume) should then be minimized with respect to the particle numbers $N_a = V_4 n_a$ and $N_m = V_4 n_m$. The resulting grand potential provides the *instanton contribution* to the pressure p and to the energy density $\epsilon = -p + T\frac{\partial p}{\partial T}$.

The general case leads to rather cumbersome resulting equations, but for $T > T_c$ one has $N_a = 0$ and the following simple result $p = \frac{b}{2}n_m$ [3].

The statistical sum under consideration describes two phases, with and without chiral symmetry. However, the realistic description should include the contributions to thermodynamics *unrelated with instantons* as well: it is the *total* pressure, which

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[3] It is interesting, that connecting n_m with the gluon condensate $\frac{\alpha_s}{\pi}\langle G_{\mu\nu}^a G_{\mu\nu}^a \rangle = 16n_m$ one reproduces the famous "trace anomaly" expression. However, that the contribution of molecules to the energy density is not just the same expression with the opposite sign, because the fermionic determinant has an explicit T-dependence.

should be continuous through the transition. We use the simplest possible model here, including the non-interacting massless pions in the broken phase (which for $N_f = 2, 3$ are actually irrelevant) and the ideal quark-gluon plasma in the symmetric phase.

A typical set of results is shown in Fig. 2 for $N_f = 2$ and the cores $\frac{R_c}{\rho} = 1$ and 2, to see uncertainties involved. The constants A and B entering the activities could be determined from first principles, provided we know the accurate value of Λ_{QCD} and instanton interactions. We select B in order to get the instanton density at $T = 0$ equal to that found on the lattice, $1.4 fm^{-4}$, and A in order to get $T_c \simeq 150 MeV$.

The upper panel of Fig. 2 tells us, that although the "random component" (solid line) dominates the broken phase, the number of molecules jumps up at the transition, and the total instanton density above T_c, $2n_m$, turns out to be comparable to that at $T = 0$. We have not plotted the behaviour of the *quark condensate*, which scales as $< \bar{\psi}\psi > \sim n_a(T)^{1/2}$. Inside uncertainties of the model it is essentially flat, till nearly T_c, which qualitatively agree with lattice data.

The importance of the molecular component is better demonstrated by the T-dependence of the pressure [4] (the middle panel of Fig. 2): in fact, for $T = (1...2)T_c$ the contribution of "molecules" (dash-dotted line) is crucial, without it one would not be able to sustain the pressure because that from quarks and gluons (the dotted line) is not large enough !

The lower panel shows the large jump in energy density at the transition. Although it is mostly due to the "liberation" of quarks and gluons, a finite part of it is also generated by the "molecules" (dash-dotted line). Such behaviour was in fact observed in lattice calculations with dynamical fermions: unlike those for the pure glue case, they show $\epsilon(T)$ which overshoots the Stefan-Boltzmann value above T_c.

Finally, let us discuss what happens at larger N_f. First of all, from (6) it is

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[4] Note that both pressure and energy density are counted not from physical, but from perturbative vacuum.

clear that a positive radius of molecules can only be obtained for $\alpha > 0, b > 2$, or $N_f < 13.5$ [13]. Furthermore, for fixed interaction (namely for core parameter R_c and κ being independent on N_f) the broken phase exists in a shrinking region of temperatures, and for $N_f \geq N_f^{upper}$ the model predicts the *unbroken* chiral symmetry even in the ground state of the theory. The vacuum structure for large N_f was studied on the lattice, and (although the question is by no means settled) indications were reported [14] that $N_f^{upper} = 7$. This number is similar to what one gets in our model.

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References

[1] C. G. Callan, R. Dashen and D. J. Gross, *Phys. Rev.* **D17**, 2717 (1978).

[2] E. Shuryak, *Nucl. Phys.* **B203**, 93 (1982); **B214**, 237 (1983).

[3] D. I. Diakonov and V. Y. Petrov, *Nucl. Phys.* **B245**, 259 ( ). E. Shuryak, *Nucl. Phys.* **B302**, 559 (1988); **B319**, 521 (1989).

   E. Shuryak and J. J. M. Verbaarschot, *Nucl. Phys.* **B341**, 1 (1990).

[4] E. Shuryak and J. J. M. Verbaarschot, *Nucl.Phys.* **B410**, 55 (1993), Stony Brook preprints SUNY-NTG-92-39,40 (1992); T. Schaefer, E. Shuryak and J. J. M. Verbaarschot, Baryonic correlation functions..., Stony Brook preprint SUNY-NTG-92-41 (1992); *Nucl.Phys.* **B**, in press.

[5] E. Shuryak, *Rev. Mod. Phys.* **65**, 1 (1993).

[6] M. C. Chu, J. M. Grandy, S. Huang and J. W. Negele, *Phys.Rev.Lett.***70**, 225 (1993); M. C. Chu and S. Huang, *Phys.Rev.* **D45**, 7 (1992);

[7] S. Huang and J. W. Negele, Talks at Lattice-93, to be published in the Proceedings *Nucl.Phys.B (Proc.Suppl.)*.

[8] E. M. Ilgenfritz and E. Shuryak, *Nucl. Phys.* **B319**, 511 (1989).

[9] E. Shuryak, *Phys.Lett.* **B79**, 135 (1978).

[10] R. D. Pisarski and L. G. Yaffe, *Phys. Lett.* **B97**, 110 (1980).

[11] A. DiGiacomo, E. Meggiolaro and H. Panagopoulos, *Phys.Lett.* **B277**, 491 (1992).

[12] E. Shuryak and J. J. M. Verbaarschot, *Nucl.Phys.* **B364**, 255 (1991).

[13] E. Shuryak, *Phys.Lett.* **B196**, 373 (1987).

[14] Y. Iwasaki, K. Kanaya, S. Sakai and T. Yoshie, *Nucl.Phys.B (Proc.Suppl)* **30**, 327 (1993); see also talks at Lattice-93, to appear in the Proceedings.

Figure Captions

Fig. 1. The ratio $\tilde{I}(N_f, T)/\tilde{I}(N_f, 0)$ of the average fermionic determinant for molecules at temperature T, normalized to its value at T=0. Numbers on the figure are N_f, the number of (massless) quark flavors. The curves marked by 1 correspond to $I(T)/I(0)$, the integral for "random" component. Solid curves are for core radius $R_c = 2\rho$, and the dashed ones are for $R_c = \rho$.

Fig. 2. Chiral restoration phase transitions for 2 massless flavors and two different core parameters R_c. The upper panel shows the T dependence of the densities n_a (solid) and n_m (dotted). In the middle panel the T dependence of the total pressure p (solid) is shown, including the contributions of the pion gas/quark-gluon plasma (dotted) and of instantons (dash-dotted). The energy density is presented in the lower panel (solid) which is modified by the instanton contribution (dash-dotted).

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